Problem Set 11.3

Integral Test

Let f be a continuous, positive, decreasing function on the interval $[1,\infty)$ and suppose that $a_n = f(n)$ for all $n = 1, 2, 3, \cdots$.

Then $\int_1^{\infty} f(x)\,dx$ converges $\Leftrightarrow$ $\sum_{n=1}^{\infty} a_n$ converges.

즉, $\int_1^{\infty} f(x)\,dx$ converges $\Rightarrow$ $\sum_{n=1}^{\infty} a_n$ converges

$\int_1^{\infty} f(x)\,dx$ diverges $\Rightarrow$ $\sum_{n=1}^{\infty} a_n$ diverges

1. The $p-$series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if

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2. Use the Integral Test to show that the given series is converges

(1) $\sum_{n=2}^{\infty} \frac{1}{n\,(\ln n)^2}$

(2) $\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$

Problem Set 11.4

(Comparison Test) Suppose that $0 \le a_n \le b_n$ for $n \ge N$.

(i) $\sum b_n$ converges $\Rightarrow$ $\sum a_n$ converges.

(ii) $\sum a_n$ diverges $\Rightarrow$ $\sum b_n$ diverges.

(Limit Comparison Test)

Suppose that $a_n \ge 0$, $b_n > 0$, and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

(i) $0 < L < \infty$ $\Rightarrow \sum a_n$ and $\sum b_n$ converge or diverge together.

(ii) $L = 0$ and $\sum b_n$ converges $\Rightarrow$ $\sum a_n$ converges.

(iii) $L = \infty$ and $\sum b_n$ diverges $\Rightarrow$ $\sum a_n$ diverges.

3. Determine whether the given series converges or diverges and give a reason for your conclusion.

(1) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n\sqrt{n}}$

(2) $\sum_{n=1}^{\infty} \frac{n}{1+n^2}$

Problem Set 11.5

(Alternating Series Test)

Let $\sum_{n=1}^{\infty} (-1)^{n+1} b_n = b_1 - b_2 + b_3 - b_4 + \cdots$ be an alternating series where $b_n > 0$. Then, $\sum_{n=1}^{\infty} (-1)^{n+1} b_n$ converges if

(i) $\lim_{n\to\infty} b_n = 0$ and (ii) $b_n \ge b_{n+1}$ for $n \ge N$.

4. Determine whether the given series converges or diverges and give a reason for your conclusion.

(1) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$

(2) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n-1}{n}$

Problem Set 11.6

(Absolute Convergence Test)

$\sum |a_n|$ converges $\Rightarrow$ $\sum a_n$ converges.

(Ratio Test)

Let $\sum a_n$ be a series with $a_n \ne 0$ and $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = L$.

(i) $L < 1$ $\Rightarrow$ $\sum a_n$ converges absolutely (hence converges).

(ii) $L > 1$ $\Rightarrow$ $\sum a_n$ diverges.

(iii) $L = 1$ $\Rightarrow$ the test is inconclusive.

(Root Test)

Let $\sum a_n$ be a series with $a_n \ne 0$ and $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$.

(i) $L < 1$ $\Rightarrow$ $\sum a_n$ converges absolutely (hence converges).

(ii) $L > 1$ $\Rightarrow$ $\sum a_n$ diverges.

(iii) $L = 1$ $\Rightarrow$ the test is inconclusive.

5. Determine whether the given series converges or diverges and give a reason for your conclusion.

(1) $\sum_{n=1}^{\infty} \frac{\sin n}{n\sqrt{n}}$

(2) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{3^n}$