Problem Set 3.11

> Let $y = f(x)$ and x change from a to $a + \triangle x$.
> **actual change** : $\triangle y = f(a + \triangle x) - f(a)$
> **differential** : $dy = f'(a)dx = f'(a)\triangle x$

1. Find $\triangle y$ and dy where $y = 2x - x^2$, $x = 2$, $\triangle x = dx = -0.3$.

> At $x = a$,
> **linear approximation** : $L(x) = f(a) + f'(a)(x - a)$

2. Find the linear approximation to the function $f(x) = \sin x$ at $x = \dfrac{\pi}{6}$.

> Approximation $f(a + \triangle x)$ where $\triangle x = dx$ using;
> **a differential** : $f(a + \triangle x) \approx f(a) + f'(a)dx$
> **a linear approximation** : $f(a + \triangle x) \approx L(a + \triangle x)$

3. Approximate $\sqrt{99.8}$.

(1) [method1] Use a differential to approximate $\sqrt{99.8}$.

(2) [method2] Use a linear approximation to approximate $\sqrt{99.8}$.

Problem Set 6.6~6.7

Definitions

> $\sin^{-1}x = y \Leftrightarrow \sin y = x$ and $-\dfrac{\pi}{2} \leq y \leq \dfrac{\pi}{2}$
>
> $\sin^{-1}(\sin x) = x$ for $-\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}$
>
> $\sin(\sin^{-1}x) = x$ for $-1 \leq x \leq 1$

4. Evaluate

(1) $\sin^{-1}\left(-\dfrac{1}{2}\right) =$

(2) $\sin^{-1}(\sin\dfrac{\pi}{9}) =$

(3) $\sin^{-1}(\sin\dfrac{8\pi}{9}) =$

Derivatives

> $D_x(\sin^{-1}x) = \dfrac{1}{\sqrt{1 - x^2}}$ $D_x(\tan^{-1}x) = \dfrac{1}{1 + x^2}$

5. Find the derivative of the function.

(1) $y = \sin^{-1}(2x + 1)$

(2) $y = \ln(\tan^{-1}x)$